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Measuring Saccade Peak Velocity Using a Low-Frequency Sampling Rate of 50 Hz

Roel Wierts, Maurice J. A. Janssen*, and Herman Kingma

***Abstract*—During the last decades, small head-mounted video eye trackers have been developed in order to record eye movements. Real-time systems—with a low sampling frequency of 50/60 Hz—are used for clinical vestibular practice, but are generally considered not to be suited for measuring fast eye movements. In this paper, it is shown that saccadic eye movements, having an amplitude of at least 5°, can, in good approximation, be considered to be bandwidth limited up to a frequency of 25–30 Hz. Using the Nyquist theorem to reconstruct saccadic eye movement signals at higher temporal resolutions, it is shown that accurate values for saccade peak velocities, recorded at 50 Hz, can be obtained, but saccade peak accelerations and decelerations cannot. In conclusion, video eye trackers sampling at 50/60 Hz are appropriate for detecting the clinical relevant saccade peak velocities in contrast to what has been stated up till now.**

***Index Terms*—Eye movements, saccade, scleral search coil (SSC), video oculography (VOG).**

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I. Introduction

Since the development of small head-mounted video eye trackers, video oculography (VOG), has gained in popularity among clinical practitioners because of three dimensional recording of eye movements, low noise level, and high spatial accuracy. The developed head-mounted VOG systems can mainly be divided into two different groups: low-frequency sampling systems (50–60 Hz [1], [2]) operating real time and high-speed sampling systems up to 400 Hz [3]. Major drawbacks of the latter are that: 1) data are processed offline after the examination is completed—patient coaching during the examination is essential or 2) the available online systems are not useful in vestibular practice—they are unstable or uncomfortable during head movements. The drawback of real-time sampling VOG systems is the low sampling frequency, making measurement of fast eye movements problematic. Especially, for the calculation of saccadic peak velocities, an important parameter in clinical practice, high sampling frequencies, are always mentioned to be necessary. In a study of Juhola *et al.* [4], it was found that in order to obtain accurate values for the maximum eye velocity of a saccade of 20°, a sampling frequency of at least 300 Hz is required.

The purpose of this study is to evaluate saccade power spectra and evaluate a technique to improve the temporal resolution of 50 Hz eye movement recordings using VOG.

II. Methods

A. Eye Movement Recordings

Three healthy subjects without any history or evidence of ophthalmologic or neurologic disorders participated in the experiment. Ages ranged from 23 to 28 years. All subjects participated on a voluntary basis after giving their informed consent.

Subjects were seated in a chair. Movement of the head was minimized using a headrest attached to the chair. After calibration, subjects were asked to visually fixate a dot of 0.5 cm, projected on a screen positioned 1 m in front of the subjects. The dot moved abruptly from side to side, forcing the subjects to make horizontal midline-crossing saccades. The angle over which the spot moved started at a small value of 5° and increased in steps of 5° up to an angle of 25° (and 28°, limited by the dimensions of the screen). For each rotation angle, three saccades to the left and right were made, resulting in a total of 36 saccades per subject.

Since saccadic eye movements are the fastest eye movements one can make, they have the highest cutoff frequency of all eye movements. Thus, a sampling frequency, capable of accurately recording saccades, is also capable of recording all other types of eye movements.

Horizontal saccadic eye movements were recorded with the skalar scleral search coil (SSC) system S3020 (Skalar Medical). The SSC signal was amplified by an analogue amplifier having a bandwidth of 200 Hz. For the SSCs, the Skalar Medical combination annulus was used. The signal was recorded at a sampling frequency of 1 kHz.

B. Nyquist Sampling Theorem

When recording a dynamic signal, the used sampling frequency f_s is of high importance, since a too low sampling frequency results in a loss of information, called aliasing. The Nyquist critical frequency f_c equals half the sampling frequency f_s [5].

The Nyquist sampling theorem states that if a continuous function $x(t)$, sampled at a sampling interval $T_s = 1/f_s$, is bandwidth limited with a maximum frequency component, $f_{\max}$, equal to or smaller than f_c, then the function $x(t)$ is completely determined by its samples $x[n]$ and is given explicitly by

$$x(t) = \sum_{n=-\infty}^{+\infty} x[n] \frac{\sin[2\pi f_c (t - nT_s)]}{\pi(t - nT_s)} \quad (1)$$

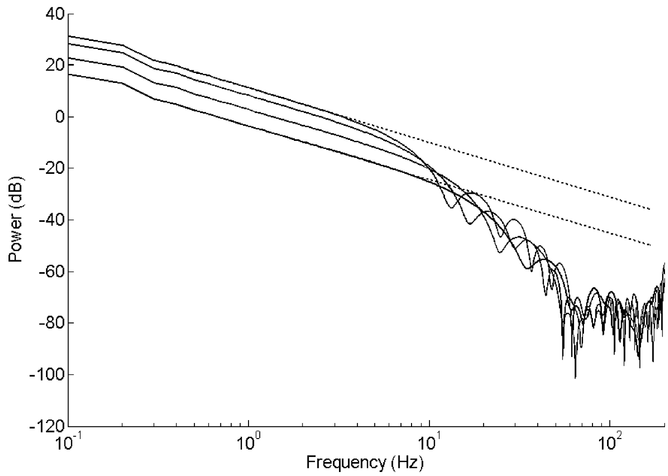

Fig. 1. Power spectra of saccades of varying amplitude. Top to bottom: 28°, 20°, 10°, and 5°. The dotted lines represent the power spectra of the idealized, instantaneous saccades with corresponding amplitudes.

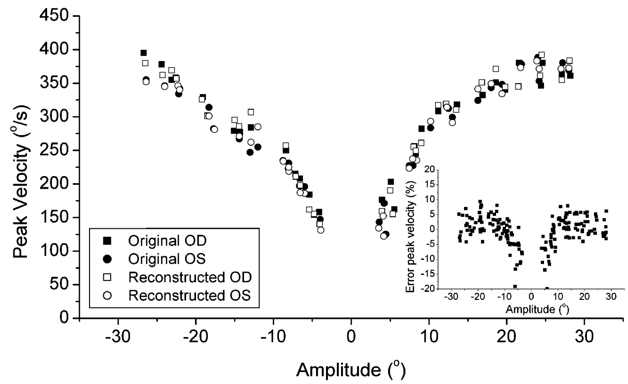

Fig. 2. Main sequence of horizontal midline-crossing saccades of the left (OS) and right (OD) eye from one subject. The filled symbols correspond to the original signal, the empty symbols with the reconstructed signal. Positive (negative) amplitudes correspond to saccades to the right (left). The inset shows the difference in peak velocity between original and reconstructed signals versus amplitude of all subjects.

with $x[n]$ given by

$$x[n] = x(nT_s), \qquad n = 0, \pm 1, \pm 2, \pm 3, \ldots. \tag{2}$$

Also, the Nyquist sampling theorem states that in order to prevent aliasing, the (continuous) signal has to be bandwidth limited with the upper frequency equal to or smaller than f_c.

The power spectra presented in this paper were obtained using a method similar to Harris *et al.* [6].

III. Results

A. Spectra of Saccades

In Fig. 1, the power spectra are shown for saccades of varying amplitude, showing that at low frequencies, there is a constant roll-off of 20 dB/decade, reflecting the overall step change in eye position [6]. The power spectra of two idealized instantaneous saccades with amplitudes of 5° and 28° are represented by the dotted lines in Fig. 1.

The fact that saccades do not occur instantaneously results in a downward departure from the theoretical 20 dB/decade roll-off at a frequency of 20–30 Hz, indicating a lack of energy of higher frequency components. Since larger amplitude saccades have longer durations, this departure begins at lower frequencies as the amplitude of the saccade increases [6]. This indicates that for the small amplitude saccades, the higher frequency components are relatively more important than for the large amplitude saccades. The lack of energy at high frequencies, together with the 20 dB/decade roll-off, indicates that for the accurate description of saccades, the high-frequency components are negligible as compared to the low-frequency components. This means that saccadic eye movements are in good approximation bandwidth limited with an upper frequency of 25–30 Hz.

B. Reconstruction Technique

As a result of this bandwidth limit, the Nyquist theorem states that a sampling frequency of 50–60 Hz is sufficiently high to avoid aliasing. Moreover, the eye position for times in between two consecutive sampling points can be obtained using (1). As a result, the original signal can be reconstructed at higher "sampling" frequencies. In order to show this, the saccadic eye position signal recorded at a frequency of 1 kHz was decimated to a frequency of 50 Hz. The obtained signal can be considered to be obtained by direct sampling of the eye position at a frequency of 50 Hz. Thereafter, the original eye position signal was reconstructed (interpolated to the original sampling frequency of 1 kHz) using (1). From each eye position signal, also the corresponding eye velocity signal was calculated using the two point central difference differentiation algorithm. Usui and Amidror [7] showed this method to be not only very practical for use, but even almost optimum. In order to avoid the differentiation of high-frequency noise, a spread of six was used [8].

In Fig. 2, the measured peak velocities of both the original and reconstructed signals for one subject are shown as a function of the saccade amplitude, the so-called main sequence [9]. From this figure, it can be seen that for saccades having amplitudes in the range of 5° to 28°, there is a good agreement between the saccade peak velocity determined for the original and reconstructed signals. For the small amplitude saccades of about 5°, an underestimation of the peak velocity of 10%–15% was found, shown in the inset of Fig. 2 for all subjects. For the larger amplitude saccades, the deviation of peak velocity was 3% on average, without a preference for under- or overestimation. Typical VOG noise of 0.05° results in an additional error of 0.5%, independent of the saccade amplitude. Furthermore, for correction saccades present in the signal, having an amplitude smaller than 5°, it was observed that the reconstruction technique often did not result in reliable position and velocity signals resulting in a large underestimation of the peak velocity. And also, ringing occurs at the start and end of the saccade, which shows that the eye acceleration and decelerations cannot be detected or reconstructed accurately with sampling rates of 50–60 Hz or less.

IV. Discussion

From the power spectra of saccades, it was shown that saccades having an amplitude of at least 5°, can be considered to be bandwidth limited with an upper frequency of 25–30 Hz. The fact that the higher frequency components still contain some relevant information results in ringing and somewhat ill-defined saccade duration and amplitude in the reconstructed signal at small amplitude saccades. Good approximations of saccade peak velocity can be obtained, as deviations of 3% are much smaller than normal variations [10].

In this study, three healthy subjects were considered. Since eye velocity is not increased in case of vestibular or ophthalmologic pathology, the reconstruction technique will also yield reliable eye velocity signals in patients.

V. Conclusion

A sampling frequency of about 50 Hz is sufficiently high to prevent aliasing. Furthermore, it was shown that using the Nyquist sampling theorem to interpolate the data to 1 kHz, accurate values for the peak velocity of saccades, recorded at 50 Hz, can be obtained. From these observations, it can be concluded that the presented reconstruction technique is a promising technique for improving low-frequency real-time VOG systems.

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Comments on "Sliding Mode Closed-Loop Control of FES: Controlling the Shank Movement"

Mir-Mahdi Ebrahimpour and Abbas Erfanian*

***Abstract*—There are some essential problems with the arguments presented in the above paper about the design of a sliding-mode controller for functional electrical simulation (FES) induced control of knee-joint angle. In this note, we show that applying some approximations in derivation of the control law violates the reaching condition and could introduce some parasitic unmodeled dynamics in the sliding-mode control loop. Therefore, the proposed controller cannot force the system into a sliding-mode regime, and its ability of producing a robust control loop with good tracking performance is theoretically under question.**

***Index Terms*—Functional electrical stimulation, sliding-mode control.**

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Recently, a sliding-mode controller (SMC) was proposed in the aforementioned paper [1] for control of knee-joint angle using functional electrical stimulation of quadriceps muscle group. The authors proposed a method to deal with the problem of immeasurable state variables in a sliding-mode control design and claimed that their novel model-based nonlinear controller is able to guarantee both robustness and good tracking performance. Controversially, an in-depth review of the proposed design criteria shows that the controller cannot force the system into a sliding-mode regime and does not ensure robustness and good tracking performance consequently. Moreover, their proposed method for bypassing the problem of immeasurable state variables is inefficient and can produce chattering in a sliding-mode control loop.

To design the controller, the authors employed a neuromuscular–skeletal model of a knee joint developed by Riener *et al*. The neuromuscular–skeletal model was translated into a state-space representation needed for designing the controller with four state variables (i.e., $x_1,\ x_2,\ x_3,\ x_4$) and was split into two cascaded subsystems. The state variable x_3 was considered as the input of subsystem 1 and as an output of subsystem 2, which was driven by the input u. The state variables x_1 and x_2 were regarded as the outputs of subsystem 1. Since the state variables x_3 and x_4 are immeasurable, the authors were not able to design a full-state-feedback sliding-mode controller. However, design of a sliding-mode controller for subsystem 1 was allowed due to the special structure of the model. Denoting the sliding variable by s, the authors replaced the discontinuous term $k\,\mathrm{sign}(s)$ with the continuous term ks in the proposed control law to reduce the effect of chattering. In the next step, they used an approximate inversion of subsystem 2 to cancel out its dynamics.

Unfortunately, erroneous arguments were utilized in the design of the sliding-mode control. A sliding-mode control law must be able to force the state values of the plant to reach to a predefined sliding surface in finite time (i.e., sliding condition). One should note that the condition $s\dot{s} < 0$ is not sufficient to ensure a finite-time convergence to the sliding surface and a stronger η-reachability condition is needed for this purpose as follows [2]:

$$s\,\dot{s} \le -\eta\,|s|, \qquad \eta > 0. \tag{1}$$

The η-reachability condition can be satisfied by adding the following corrective control to the equivalent control [3]

$$u_c = -k\,\mathrm{sign}(s), \qquad k > 0. \tag{2}$$

Consider the following second-order nonlinear system

$$\ddot{x} = f(x,\,t) + u(t) \tag{3}$$

where $f(x,\,t)$ denotes an unknown bounded nonlinear function, $x(t)$ is the state variable, and $u(t)$ is the system input. In the design of a sliding-mode controller to force the state variable $x(t)$ of system (3) to track a desired trajectory $x_d(t)$, the conventional definition of the sliding surface is

$$s = \dot{e} + \lambda e \tag{4}$$

where e denotes instantaneous tracking error. Using (3), differentiation of (4) with respect to time can be written as

$$\dot{s} = \ddot{e} + \lambda\,\dot{e} = \ddot{x} - \ddot{x}_d + \lambda\,\dot{e} = u + f - \ddot{x}_d + \lambda\,\dot{e}. \tag{5}$$

The equivalent control can be obtained by setting $\dot{s} = 0$ as

$$u_{\mathrm{eq}} = -f + \ddot{x}_d - \lambda\,\dot{e}. \tag{6}$$

The first-order sliding-mode control law is

$$u = \hat{u}_{\mathrm{eq}} - k\,\mathrm{sign}(s), \qquad k > 0 \tag{7}$$